Indian Statistical Institute, Bangalore

B.Math (Hons.)II Year, First Semester

Mid-Sem Examination

Algebra -III

Time: 3 hours Date:27 Sept 2010 Instructor: N.S.N.Sastry

Maximum Marks 100

Answer all questions. Your arguments should be complete and to the point.

1. Let R be a ring. Define an indecomposable $R-$ module and an irreducible $R-$ module. Give an example of an indecomposable $R-$ module which is not an irreducible $R-$ module. [2 + 2 + 4]

2. Let $R = M_n(\mathbb{R})$, the ring of $n \times n$ matrices $(n \geq 1)$ and with real entries $A \in R$ be the matrix with only one nonzero entry. Determine the minimal left ideal of R containing A and the minimal right ideal of R containing A.

   [8 + 8]

3. Define the localization R_P of a commutative ring R (with 1) at a prime ideal P. Show that R_P has a unique maximal ideal. [6 + 4]

4. a) Determine the number of abelian groups of order 2592, up to isomorphism. State precisely the results you use.

   b) Show that the multiplicative group of the non-zero elements of a finite field is cyclic. [8 + 8]

5. Show that, given any $n \times n$ matrix A over a field F, there exists a nonsingular matrix P over F of order n such that $PAP^{-1} = D + N$, where D is a diagonal matrix and N is a nilpotent matrix and $DN = ND$. State precisely the result you use. [16]

6. Find an integer x such that $x \equiv 3 \pmod 7), x \equiv 3 \pmod 9)$ and $x \equiv 4 \pmod 5)$. [10]

7. Justify your answers:

   a) Give an example of a maximal ideal in $\mathbb{Z}[X, Y]$.

   b) Give an example of a subring of $\mathbb{R}$ in which 2 is invertible, but not 3.

   c) Construct a field consisting of 4 elements. [7 + 6 + 7]

——————————end——————————